Naval Research Laboratory
Washington, DC 20375-5320

NRL/MR/6750--08-9122

Preliminary Global Radiation Belt Formation and Prediction Model

MICHAEL J. KESKINEN

Charged Particle Physics Branch
Plasma Physics Division

June 25, 2008

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
25-06-2008

2. REPORT TYPE
Memorandum Report

3. DATES COVERED *(From - To)*

4. TITLE AND SUBTITLE
Preliminary Global Radiation Belt Formation and Prediction Model

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Michael J. Keskinen

5d. PROJECT NUMBER
67-9387-F7

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375-5320

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL/MR/6750--08-9122

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research
One Liberty Center
875 North Randolph St.
Arlington, VA 22203-1995

10. SPONSOR / MONITOR'S ACRONYM(S)
ONR

11. SPONSOR / MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
A computational model for the time-dependent dynamics of the electron and ion terrestrial radiation belts has been developed. The model solves the fundamental bounce-averaged electron and ion modified Boltzmann Fokker-Planck equations for energy, pitch angle, and L-shell. The code has been applied to broadband whistler turbulence and found to be consistent with empirical models of radiation belt dynamics.

15. SUBJECT TERMS
Radiation belt
Space weather
Satellite damage

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	UL	26	Michael J. Keskinen

19b. TELEPHONE NUMBER *(include area code)*
(202) 767-3215

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

1. Introduction

The earth's radiation belts are composed of approximately tens of keV to several MeV electrons and hundreds to several hundred MeV ions which are trapped in the magnetosphere from approximately $1.2 < \mathrm{L} < 8$. Depending on energy, the electron population typically reside in two distinct inner and outer belt regions separated by the slot region ($1.8 < \mathrm{L} < 3.2$).

The terrestrial radiation belts can have a significant effects on space systems and space technology. Moderate energy (10-100 keV) electrons can lead to surface charging in space systems while relativistic MeV electrons can cause deep-dielectric charging in spececraft materials. During geomagnetic substorms, large fluctuations in electron differential flux can occur on relatively short time scales. A time-dependent predictive model for the earth's radiation belts will have both scientific and practical importance.

Trapped charged particles in the earth's radiation belts have multiple-periodic motions on three distinct timescales. To describe this motion [*Northrop*, 1963; *Roederer*, 1970] three adiabatic invariants, i.e, J_1, J_2, and J_3, have been developed with $\mathrm{J}_1 = \pi \mathrm{cp}^2 \sin^2\alpha/2\mathrm{qB}$, $\mathrm{J}_2 = \int \mathrm{p}_{\|} \mathrm{ds}$, $\mathrm{J}_3 = (\mathrm{q}/\mathrm{c})\Phi$. Here, α is the pitch angle, p is the total momentum, $\mathrm{p}_{\|}$ is the component of momentum parallel to the geomagnetic field, and Φ is the magnetic flux enclosed through the drift shell. An alternative set of invariants are M, $\mathrm{J} = \mathrm{J}_2$, and Φ with $\mathrm{M} = \mathrm{p}^2 \sin^2\alpha/2\mathrm{m}_0\mathrm{B}$ with m_0 the rest mass.

Manuscript approved June 5, 2008.

When turbulence, e.g., from wave particle interactions, produce fluctuations in the J's that are small, random, and high frequency, the changes in f can be modeled as a stochastic diffusive process and can be described by a Fokker-Planck formalism. In this formalism, the description of radiation belt dynamics involves a balance between diffusive transport, convection, sources, and losses. During magnetic storms, external sources of both electrons and ions are important and need to be incorporated. Key sources of phase space transport include radial (L) diffusion due to fluctuations in the convection electric field and magnetospheric magnetic field, pitch angle and energy diffusion, e.g., due to whistler waves, pitch angle diffusion and energy losses from collisional processes involving cold plasmaspheric plasma and neutral atmospheric species, and charge exchange losses. Precipitation losses can occur when the pitch angle is inside the loss cone.

Several models have been developed to simulate global radiation belt dynamics. Two different approaches have been taken, i.e., the test-particle and kinetic formulations. The test particle approach involves the tracing of individual particle orbits in the combined electric and magnetic fields derived from an MHD model of the magnetosphere. Using the test particle method, guiding center simulations [*Li et al.*,, 1993; *Hudson et al.*, 1996] of the sudden storm commencement induced by the interplanetary shock of March 24, 1991 were performed and analyzed. These studies suggested the formation of a new electron belt at L=2.5. In addition, test particle simulations of the outer belt electrons [*Ukhorskiy et al.*, 2006] were performed to model the magnetic storm of September 7, 2002.

Kinetic models have also been used to model electron radiation belt dynamics. This approach solves the modified Boltzmann Fokker-Planck equation with wave-induced and collisional pitch angle and energy diffusion coupled with particle sources and losses. Radial diffusion, combined with acceleration and loss due to whistler mode VLF chorus, indicated that [*Varotsou et al.*, 2005; *Horne et al.*, 2006] wave acceleration by whistler waves is an important mechanism for the outer radiation belt electrons. *Beutier and Boscher* [1995] and *Bourdarie et al.* [1997] solved for a phase space three-dimensional model of the electron radiation belt. They found that cosmic ray albedo neutron decay internal sources are not sufficient to account for the typical quiet time electron fluxes. A convection-diffusion model [*Fok et al.*, 2001; *Zheng et al.*, 2003; *Fok et al.*, 2008] has been developed to simulate electron radiation belt dynamics. The model was used to simulate a substorm injection during a dipolarization of the magnetic field during a substorm. Experimentally observable features such as drift echoes were successfully reproduced.

In this paper we develop a first principles time-dependent simulation model for the electron and ion radiation belt dynamics. This is accomplished by solving the convection-diffusion model of radiation belt dynamics as derived from a modified Boltzmann Fokker-Planck model. The model includes both pitch angle and energy diffusion from both wave-particle and collisional effects, energy loss from plasmaspheric plasma and neutral atmospheric constituents, radial diffusion, and charge exchange losses. We find that the general characteristics of the electron fluxes predicted by the model are consistent with empirical models derived from satellite observations.

The outline of the paper is as follows. In Section 2 we present the time-dependent radiation belt simulation model. In section 3 we apply the model to whistler wave turbulence and compare the model output with empirical satellite models. Finally in section 4 we summarize our results.

2. Model

Using kinetic theory, for small, randomly phased fluctuations in the three adiabatic invariants, J_1, J_2, and J_3, the time dependence of the electron and ion phase space density f_k, k=e,i, can be written in the form of a Fokker-Planck equation:

$$\frac{df_k}{dt} + \sum_{i=1}^{3} \frac{\partial}{\partial J_i} G_i f_k = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{\partial}{\partial J_i} D_{ij} \frac{\partial f_k}{\partial J_j} + S_k - L_k \tag{1}$$

where G models energy loss, D describes diffusive processes, and S_k and L_k represent explicit electron and ion sources and losses within the phase space volume. In analogy with a Fokker-Planck model, G represents a nonstochastic friction and D a stochastic diffusion. The evaluation of the diffusion tensor elements is of crucial importance in the use of Eq. (1) for the modeling of the radiation belts. For pitch angle and energy diffusion, both wave-particle and low altitude collisional effects will be important while for spatial diffusion, e.g., radial diffusion, slow time scale electric and magnetic fluctuations will be needed.

The standard approach for studying the effects of experimentally observed broadband wave turbulence on the adiabatic invariants M,J,Φ is to model the effects of wave-particle interactions on pitch angle and energy diffusion using quasilinear theory [*Kennel and Petschek*, 1966]. This theory was applied to electron pitch angle scattering by whistler turbulence by *Lyons et al.* [1972] and *Lyons and Thorne* [1973]. These studies established cyclotron-resonant whistlers as a key mechanism for electron loss in the slot region and for maintaining the steady state electron distribution in combination with radial diffusion and Coulomb collisional effects at low altitudes.

In this study we include only radiation belt electron dynamics. The ion dynamics is also governed by a similar Fokker-Planck equation with different pitch angle diffusion coefficients along with different source and loss terms.

It is convenient to transform Eq. (1) from (M, J, Φ) space to (E, x, L) space where E is the energy, x=cos α_0 with α_0 the equatorial pitch angle, and L denotes L shell. Since we will be interested in time scales much longer than the electron bounce time, we bounce-average Eq.(1) for the electrons:

$$\frac{\partial\langle f\rangle}{\partial t} + \frac{dL}{dt}\frac{\partial\langle f\rangle}{\partial L} + \frac{d\phi}{dt}\frac{\partial\langle f\rangle}{\partial \phi} + \frac{1}{\gamma p}\frac{\partial}{\partial E}\gamma p\langle\frac{dE}{dt}\rangle\langle f\rangle = \frac{1}{xT\left(y\right)}\frac{\partial}{\partial x}xT\left(y\right)\langle D_{xx}\rangle\frac{\partial\langle f\rangle}{\partial x}$$

$$+\frac{1}{xT\left(y\right)}\frac{\partial}{\partial x}xT\left(y\right)\langle D_{xE}\rangle\frac{\partial\langle f\rangle}{\partial E}+\frac{1}{\gamma p}\frac{\partial}{\partial E}\gamma p\langle D_{Ex}\rangle\frac{\partial\langle f\rangle}{\partial x}$$
$$+\frac{1}{\gamma p}\frac{\partial}{\partial E}\gamma p\langle D_{EE}\rangle\frac{\partial\langle f\rangle}{\partial E}+L^2\frac{\partial}{\partial L}\frac{1}{L^2}D_{LL}\frac{\partial\langle f\rangle}{\partial L}+S_e-L_e \quad (2)$$

In Eq. (2), the angle brackets denote bounce averaging, i.e., applying the operator $\tau_{\mathrm{B}}\int \mathrm{ds}/\mathrm{v}_{\|}$ with τ_B the bounce period and the coordinate s denotes distance along the geomagnetic field. In Eq. (2), the second and third terms on the left hand side represent radial and azimuthal convection, the fourth term represents electron energy loss due to collisional effects in the plasmasphere and atmosphere, the first term on the right hand side represents pitch angle diffusion from both collisional effects and wave-particle interactions, the second and third terms represent cross energy and pitch angle diffusion, the fourth term on the right hand side represents energy diffusion from both collisional and wave-particle effects, and the fifth term gives radial L-diffusion. In addition, we have defined $\mathrm{y} = (1-\mathrm{x}^2)^{1/2} = \sin\alpha_0$, and $\mathrm{T(y)} = 1.38 - 0.31(\mathrm{y}+\mathrm{y}^{1/2})$.

The energy loss rate for the radiation belt electrons comes primarily from inelastic collisions with neutral atmospheric atoms and elastic collisions with plasmaspheric and ionospheric electrons. The bounce-averaged energy loss rate for electrons can be written [*Walt and MacDonald*, 1964]:

$$\langle\frac{dE}{dt}\rangle = -\frac{4\pi e^4}{mcT\left(y\right)}\frac{E+E_0}{E^{\frac{1}{2}}\left(E+2E_0\right)^{\frac{1}{2}}}\int_0^{\lambda_m} d\lambda\frac{sin^2\alpha\, cos^7\lambda}{cos\alpha\, sin^2\alpha_0}f\left(n_e,n,E\right) \quad (3)$$

with

$$f\left(n_e, n, E\right) = n_e ln\eta_D^{-1} + \sum_i Z_i n_i \left(ln\frac{E\left(\frac{E}{E_0}+2\right)^{\frac{1}{2}}}{I_i} - \frac{1}{2}\frac{E}{E+E_0} \right) \tag{4}$$

In Eq. (3), α denotes the local pitch angle, λ is the magnetic latitude, λ_m is the magnetic mirror latitude, $\mathrm{E_0} = \mathrm{mc}^2$, I_i is the ionization energy, $\eta_\mathrm{D} = \hbar/\mathrm{m_r v}\lambda_\mathrm{D}$, m_r is the reduced mass, $\mathrm{n_e}$ is the electron density, n_i is the neutral density of species i, and Z is the atomic number. The electron and ion densities H^+, He^+, O^+, O_2^+, NO^+ are found using the International Reference Ionosphere [*Bilitza*, 2000] combined with a plasmaspheric model [*Chiu et al.*, 1979]. The neutral species are found using the MSIS90 model[*Hedin*, 1991].

The bounce-averaged pitch angle diffusion coefficient $\langle \mathrm{D_{xx}} \rangle$ in Eq. (2) is composed of a collisional part and wave-particle interaction part and can be written $\mathrm{D_{xx}} = \mathrm{D^c_{xx}} + \mathrm{D^w_{xx}}$. Similarly, the bounce-averaged energy diffusion coefficient $\langle \mathrm{D_{EE}} \rangle$ is composed of a collisional part and wave-particle interaction part and can be written $\mathrm{D_{EE}} = \mathrm{D^c_{EE}} + \mathrm{D^w_{EE}}$. The cross tensor $\mathrm{D_{xE}}$ and $\mathrm{D_{Ex}}$ elements are produced primarily by wave-particle interactions. The bounce-averaged collisional pitch angle scattering diffusion coefficient can be written [*Walt and MacDonald*, 1964]:

$$\langle D^c_{xx} \rangle = \frac{2\pi e^4 c\left(E+E_0\right)}{E^{\frac{3}{2}}\left(E+2E_0\right)^{\frac{3}{2}} T\left(y\right)} \int_0^{\lambda_m} d\lambda \frac{cos\alpha\, cos^7\lambda}{cos^2\alpha_0} g\left(n_e, n, E\right) \tag{5}$$

with

$$g\left(n_e, n, E\right) = n_p ln\eta_D^{-1} + n_e ln\eta_D^{-1} + \sum_i Z_i^2 n_i ln\eta_i^{-1} \tag{6}$$

$\eta_i = \mathrm{Z}^{1/3}(1-\beta^2)^{1/2}/137\beta$, and $\beta = \mathrm{v/c}$. The collisional energy diffusion coefficient is typically much smaller than the collisional pitch angle diffusion coefficient and will not be considered further.

In order to estimate the wave-particle contribution to pitch angle and energy diffusion, a specific wave-particle interaction must be specified. Wave-particle interactions play a fundamental role in radiation belt electron dynamics. Gyroresonant interactions are produced by the local Doppler-shifted cyclotron resonance condition, $\omega - \mathrm{s}\Omega_e/\gamma - \mathrm{kv}\cos\alpha = 0$. Here s=1 denotes right hand R-mode waves and s=-1 the left hand L-mode waves and γ is the Lorentz factor. Gyroresonant interactions can occur with both high and low frequency waves in the range of approximately $0.1\Omega_{\mathrm{O}^+} < \omega < 0.8\Omega_{\mathrm{e}}$ where Ω_{O^+} is the oxygen ion gyrofrequency and Ω_{e} is the electron gyrofrequency. Waves in this frequency range include whistler mode chorus, plasmaspheric hiss, and electromagnetic ion cyclotron (EMIC) waves.

In this study we consider only R-mode whistler turbulence. The evolution of f in the presence of whistler wave turbulence can be found from the standard quasilinear theory [*Lyons et al.*, 1974]:

$$\frac{\partial f}{\partial t} = \frac{1}{sin\alpha}\frac{\partial}{\partial\alpha}D_{\alpha\alpha}sin\alpha\frac{\partial f}{\partial\alpha} + \frac{1}{sin\alpha}\frac{\partial}{\partial\alpha}D_{\alpha p}sin\alpha\frac{\partial f}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p}p^2 D_{p\alpha}\frac{\partial f}{\partial\alpha} + \frac{1}{p^2}\frac{\partial}{\partial p}p^2 D_{pp}\frac{\partial f}{\partial p} \qquad (7)$$

For parallel propagating R-mode whistler waves the pitch angle diffusion coefficient can be written [*Summers*, 2005]:

$$D_{\alpha\alpha} = \frac{\pi}{2}\frac{\Omega_e^2}{B_0^2}\frac{E_0^2}{(E+E_0)^2}\sum_i \frac{\left(1-\frac{\omega_i}{k_i v}cos\alpha\right)^2}{(v\, cos\alpha - v_g)} I\left(k_i\right) \tag{8}$$

The cross-diffusion coefficient can be written:

$$D_{\alpha p} = -\frac{\pi}{2}\frac{\Omega_e p}{B_0^2}\frac{E_0^2 sin\alpha}{(E+E_0)^2}\sum_i \frac{\left(\frac{\omega_i}{k_i v}\right)\left(1-\frac{\omega_i}{k_i v}cos\alpha\right)}{(vcos\alpha - v_g)} I\left(k_i\right) \tag{9}$$

The energy diffusion coefficient can be written:

$$D_{pp} = \frac{\pi}{2}\frac{\Omega_e^2 p^2}{B_0^2}\frac{E_0^2 sin^2\alpha}{(E+E_0)^2}\sum_i \frac{\left(\frac{\omega_i}{k_i v}\right)^2}{(vcos\alpha - v_g)} I\left(k_i\right) \tag{10}$$

In Eq. (8)-(10) the ω_i and k_i satisfy both the cyclotron resonance condition with $k = k_{\parallel}$

$$\omega_i - k_i\, v\, cos\alpha - \frac{\Omega_e}{\gamma} = 0 \tag{11}$$

and the cold plasma linear dispersion relation:

$$\frac{c^2 k_i^2}{\omega_i^2} = 1 - \frac{\omega_{pe}^2}{(\omega_i - \Omega_e)(\omega_i + \Omega_i)} \tag{12}$$

The whistler wave spectrum I(k) in Eq. (8)-(10) is defined by:

$$\frac{B_1^2}{8\pi} = \int_{k_1}^{k_2} I\left(k\right) dk \tag{13}$$

where the total magnetic field $B = B_0 + B_1$ is composed of a background part B_0 and a small fluctuating part B_1.

Including whistler wave convection, the quasilinear equation for the whistler wave spectrum I can be written:

$$\frac{\partial I}{\partial t} + \frac{\partial \omega}{\partial \mathbf{k}} \frac{\partial I}{\partial \mathbf{r}} - \frac{\partial \omega}{\partial \mathbf{r}} \frac{\partial I}{\partial \mathbf{k}} = 2\gamma\left(k\right) I\left(k\right) \tag{14}$$

where $\mathrm{v}_g = \partial\omega/\partial\mathrm{k}$ is the whistler group velocity and the the growth rate γ given by [*Kennel and Petschek*, 1966]:

$$\gamma = 2\pi^2\Omega_e \left(1 - \frac{\omega}{\Omega_e}\right)^3 v_R \eta\left(v_R\right) \left[A\left(v_R\right) - \frac{1}{\frac{\Omega_e}{\omega} - 1}\right] \tag{15}$$

with

$$\eta\left(v_R\right) = 2\pi v_R \int_0^\infty dv_\perp v_\perp f\left(v_\perp, v_\parallel = v_R\right) \tag{16}$$

and

$$A\left(v_R\right) = \frac{1}{\eta\left(v_R\right)} \int_0^\infty dv_\perp \frac{v_\perp^2}{v_\parallel} \left(v_\parallel \frac{\partial f}{\partial v_\perp} - v_\perp \frac{\partial f}{\partial v_\parallel}\right) \Big|_{v_\parallel = v_R} \tag{17}$$

Here $\mathrm{v_R}$ is the resonant velocity determined from Eq. (11).

Transforming from $(\mathrm{v}_\perp, \mathrm{v}_\parallel)$ coordinates to energy and pitch angle coordinates (E, α) the growth rate can be written:

$$\gamma = 2\pi^2\Omega_e \left(1 - \frac{\omega}{\Omega_e}\right)^3 \left(\frac{2E_R}{m_e}\right)^{\frac{3}{2}} \frac{E_R + E_0}{E_0} \zeta\left(E_R\right) \left[A_E\left(E_R\right) - \frac{1}{\frac{\Omega_e}{\omega} - 1}\right] \tag{18}$$

with

$$\zeta\left(E_R\right) = \int d\alpha \, tan\alpha \, f\left(\alpha, E_R\right) \tag{19}$$

and

$$A_E\left(E_R\right) = \frac{1}{\zeta\left(E_R\right)} \frac{2E_R E_0}{E_R\left(E_R + 2E_0\right)} \int d\alpha \frac{tan^2\alpha}{cos^2\alpha} \frac{\partial f\left(\alpha, E_R\right)}{\partial \alpha} \tag{20}$$

where $\mathrm{E_R} = (1/2)\mathrm{m}_e\mathrm{v_R^2}$.

Consistent with experimental observations, we consider a whistler wave spectrum of the form:

$$I(\omega) = \frac{B_1^2}{8\pi}\frac{1}{\rho\delta\omega}e^{-\left(\frac{\omega-\omega_m}{\delta\omega}\right)^2} \tag{21}$$

with

$$\rho = \frac{\pi^{\frac{1}{2}}}{2}\left[erf\left(\frac{\omega_m-\omega_2}{\delta\omega}\right)+erf\left(\frac{\omega_2-\omega_m}{\delta\omega}\right)\right] \tag{22}$$

Here $\mathrm{B_1}$ is the rms wave magnetic field, ω_1 is the lower frequency limit to the wave spectrum, ω_2 is the upper frequency limit to the wave spectrum, ω_m is the center frequency, $\delta\omega$ is the frequency bandwidth and erf is the error function. To match typical experimental observations, $\omega_1 = 0.05\Omega_e$, $\omega_2 = 0.65\Omega_e$, $\omega_m = 0.35\Omega_e$, $\delta\omega = 0.15\Omega_e$, $\mathrm{B_1} = 100\mathrm{pT}$, and $\omega_e/\Omega_e = 2.5$. We have $\mathrm{I}(\omega) = (2\pi/\mathrm{v}_g)\mathrm{I}(k)$. Using the form for the wave spectrum in Eq. (21)-(22) we can write Eq. (8)-(10):

$$D_{\alpha\alpha} = \frac{\pi}{2}\frac{\Omega_e^2}{\rho}\frac{E_0^2}{(E+E_0)^2}\sum_i\left(\frac{B_1}{B_0}\right)^2\frac{\left(1-\frac{\omega_i}{k_i v}cos\alpha\right)^2 v_g}{\delta\omega\,(v\,cos\alpha - v_g)}e^{-\left(\frac{\omega_i-\omega_m}{\delta\omega}\right)^2} \tag{23}$$

The cross-diffusion coefficient can be written:

$$D_{\alpha p} = -\frac{\pi}{2}\frac{\Omega_e^2 p}{\beta\rho}\frac{E_0^2 sin\alpha}{(E+E_0)^2}\sum_i\left(\frac{B_1}{B_0}\right)^2\frac{\left(\frac{\omega_i}{k_i v}cos\alpha\right) v_g}{\delta\omega\,(vcos\alpha - v_g)}e^{-\left(\frac{\omega_i-\omega_m}{\delta\omega}\right)^2} \tag{24}$$

The energy diffusion coefficient can be written:

$$D_{pp} = \frac{\pi}{2}\frac{\Omega_e^2 p^2}{\beta^2\rho}\frac{E_0^2 sin^2\alpha}{(E+E_0)^2}\sum_i\left(\frac{B_1}{B_0}\right)^2\frac{\left(\frac{\omega_i}{k_i v}cos\alpha\right)^2 v_g}{\delta\omega\,(vcos\alpha - v_g)}e^{-\left(\frac{\omega_i-\omega_m}{\delta\omega}\right)^2} \tag{25}$$

For inclusion into Eq. (2), the bounce-averaged pitch angle diffusion coefficient can then be written:

$$\langle D^{w}_{\alpha\alpha}\rangle = \frac{1}{T\left(y\right)}\int_{0}^{\lambda_m}\frac{cos\alpha\, cos^7\lambda}{cos^2\alpha_0}D_{\alpha\alpha}d\lambda \tag{26}$$

In addition, the bounce-averaged energy diffusion coefficient can be written:

$$\langle D^{w}_{pp}\rangle = \frac{1}{T\left(y\right)}\int_{0}^{\lambda_m}\frac{sin^2\alpha\, cos^7\lambda}{sin^2\alpha_0\, cos\alpha}D_{pp}d\lambda \tag{27}$$

The bounce-averaged cross diffusion coefficient can be written:

$$\langle D^{w}_{\alpha p}\rangle = \frac{1}{T\left(y\right)}\int_{0}^{\lambda_m}\frac{sin\alpha\, cos^7\lambda}{sin\alpha_0\, cos\alpha_0}D_{\alpha p}d\lambda \tag{28}$$

Including whistler wave convection but ignoring inhomogeneous plasma effects, the quasilinear equation for the bounce-averaged whistler spectrum I in Eq. (14) can be written:

$$\frac{\partial\langle I\rangle}{\partial t} + \gamma_g\langle I\rangle = 2\gamma\left(k\right)\langle I\rangle \tag{29}$$

where $\gamma_g = \tau_g^{-1}\mathrm{ln}(1/\mathrm{R})$, $\tau_g = \mathrm{LR}_e/\mathrm{v}_g$, and R is the ionospheric reflection coefficient. For perfect reflection, R=1 while for strong ionospheric absorption, R $<<$ 1. Here γ_g models wave convection effects out of the magnetic flux tube.

In Eq. (2) we have:

$$\langle D_{EE}\rangle = \frac{c^2\left(E+2E_0\right)}{\left(E+E_0\right)^2}\langle D_{pp}\rangle \tag{30}$$

and

$$\langle D_{\alpha E}\rangle = \frac{c^2 E}{E+E_0}\langle D_{\alpha p}\rangle \tag{31}$$

For the convection terms in Eq. (2) we assume that we can write the total drift velocity as

$$\mathbf{V}_d = \frac{\pi p^2 c}{m B^3 e}\left(1 + 2\, ctn^2\alpha\right) \mathbf{B} \times \nabla B + \frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{32}$$

with $\mathbf{E} = \mathbf{E}_{cor} + \mathbf{E}_{conv}$ with $\mathbf{E}_{cor}$ and $\mathbf{E}_{conv}$ the corotation and convection electric fields, respectively. We can then write:

$$\frac{d\phi}{dt} = \frac{-6\pi p^2 c}{m L^2 R_e^2 e B}\frac{D\left(y\right)}{y^2 T\left(y\right)} + \Omega_{cor} + \left(\frac{d\phi}{dt}\right)_{conv} \tag{33}$$

and

$$\frac{dL}{dt} = \left(\frac{dL}{dt}\right)_{conv} \tag{34}$$

with $\mathrm{D(y) = 0.45 - 0.19y - 0.106y^{3/4}}$.

3. Results

For this preliminary study, we solve Eq. (2) numerically including only energy loss, collisional pitch angle diffusion, and whistler wave pitch angle diffusion together with a source and loss term. Radial and azimuthal convection as well as radial diffusion effects are not included. The total pitch angle diffusion coefficient is the sum of Eq.(5) and Eq.(26). For the source term we use $\mathrm{S}_e = (\mathrm{m}_e/2\mathrm{E}_0)^{3/2}(1/\mathrm{n}_0)(\mathrm{dn}_e/\mathrm{dt})\sin^2\alpha_0\exp(-\mathrm{E}/\mathrm{E}_0)$. For the loss term we take $\mathrm{L}_e = 4\mathrm{g}(\alpha_0)\langle \mathrm{f}\rangle/\tau_B$ with $\mathrm{g}(\alpha_0) = 0$ for pitch angles outside the loss cone, i.e., $\alpha_0 > \alpha_\mathrm{L}$ and $\mathrm{g}(\alpha_0) = 1$ for pitch angles inside the loss cone, i.e., $\alpha_0 < \alpha_\mathrm{L}$. We treat the range of L-shell, pitch angle, and energy of $1 < \mathrm{L} < 7$, $3^o < \alpha < 90^o$, and

$0.1\,\mathrm{MeV} < \mathrm{E} < \ 5\mathrm{MeV}$.

Fig. 1 shows the bounce-averaged pitch angle diffusion coefficient Eq. (26) as a function of equatorial pitch angle for several energies. Here we have used the whistler wave spectrum Eq. (21) for parallel propagating whistler turbulence with mean wave amplitude of B=100 pT situated at L=4.5. The pitch-angle diffusion coefficient for the MeV electrons is smaller compared to the smaller energies.

Fig. 2 gives an example of the omnidirectional differential electron flux at a fixed energy of 0.5 MeV in the meridional plane.

Fig. 3 displays the energy spectrum of the electron differential flux at L=1.2 and compares it with the typical energy spectrum from the NASA AE8MIN empirical model [*Vette*, 1991]. The energy spectrum scales approximately as $\mathrm{E}^{-\delta}$ with $\delta \simeq 2-4$ depending on the energy range.

Fig. 4 shows the L-shell dependence of the differential flux at a fixed energy of 1.5 MeV for both the physical model and the NASA model. Here the flux shows a broad peak at approximately L=4 in agreement with the NASA AE8MIN empirical model.

Fig. 5 gives the dependence of the differential flux on equatorial pitch angle at L=4 and at a fixed energy of 1.5 MeV for the model and NASA empirical model. The loss cone is evident at small pitch angles and is consistent with the NASA empirical model.

4. Summary

We have developed a first-principles time-dependent model for the electron and ion radiation belts. The model consists of collisional energy loss, collisional pitch angle scattering, wave-particle induced pitch angle scattering, energy diffusion, cross energy-pitch angle diffusiion, radial L convection, azimuthal (local time) convection, and radial diffusive effects. The model equations are in the form of a modified Boltzmann Fokker-Planck model and treat the range of L-shell, pitch angle, and energy of $1 < \mathrm{L} < 7$, $3^o < \alpha < 90^o$, and $0.1\,\mathrm{MeV} < \mathrm{E} < 5\mathrm{MeV}$.

The model has been applied to parallel propagating R-mode whistler turbulence. Quantities derived from these initial studies, i.e., omnidirectional electron flux, energy spectra, L-shell electron flux dependence, pitch angle diffusion as a function of pitch angle compare favorably with those derived from the NASA empirical model AE8MIN.

In the future we will consider other wave-induced pitch angle and energy diffusion sources, e.g., Alfven waves and electromagnetic ion cyclotron waves.

Acknowledgments

We thank Guru Ganguli for useful discussions. This work was supported by the Office of Naval Research.

References

Albert, J., Comparison of pitch angle diffusion by turbulent and monochromatic whistler waves, *J. Geophys. Res., 106*, 8477, 2001.

Beutier, T.,and D. Boscher, A three-dimensional analysis of the electron radiation belt, *J. Geophys. Res., 100*, 14853, 1995.

Bilitza, D., International Reference Ionosphere 2000, *Radio Sci., 36*, 261, 2001.

Bourdarie, S., D. Boscher, T. Beutier, J.A. Sauvaud, and M. Blanc, Electron and proton radiation belt dynamic simulations during storm periods: A new asymmetric convection diffusion model, *J. Geophys. Res., 102*, 17541, 1997.

Chiu, Y.T., J.G. Luhmann, B.K. Ching, and D.J. Boucher, An equilibrium model of plasmaspheric composition and density, *J. Geophys. Res., 84*, 909, 1979.

Fok, M.C, T.E. Moore, and W.N. Spjeldvik (2001), Rapid enhancement of radiation belt electron fluxes due to substorm diploarization of the geomagnetic field, *J. Geophys. Res., 106*, 3873.

Fok, M.C, R.B. Horne, N.P. Meredith, and S.A. Glauert(2008), Radiation belt environment model: application to space weather nowcasting, *J. Geophys. Res., 113*, A03S08, doi:10.1029/2007JA012558.

Hedin, A.E., Extension of the MSIS thermospheric model into the middle and lower atmosphere, *J. Geophys. Res., 96*, 1159, 1991.

Horne, R.B., N.P. Meredith, S.A. Glauert, A. Varotsou, D. Boscher, R.M. Thorne, Y.Y. Shprits, and R.R. Anderson (2006), Mechanisms for the acceleration of radiation belt electrons, in *Recurrent Magnetic Storms: Corotating Solar Wind Streams, Geophys. Monogr. Ser*, vol. 167, ed. B.T. Tsurutani et al, p. 151, AGU, Washington, DC.

Hudson, M.K., S.R. Elkington, J.G. Lyon, V.A. Marchenko, I. Roth, M. Temerin, and M.S. Gussenhoven (1996), MHD/particle simulation of radiation belt formation during a sudden storm commencement, in *Radiation Belts, Models and Standards, Geophys. Monogr. Ser.*, vol. 97, ed. J.F. Lemaire, D. Heyndercks, and D.N. Baker, p. 57, AGU, Washington, DC.

Kennel, C.F., and H.E. Petschek (1966), Limit on stably trapped particle fluxes, *J. Geophys. Res., 71*, 1.

Li, X., I. Roth, M. Temerin, J. Wygant, M.K. Hudson, and J.B. Blake (1993), Simulation of the prompt energization and transport of radiation particles during the March 23, 1991 SSC, *Geophys. Res. Lett., 20*, 2423.

Lyons, L.R., R.M. Thorne, and C.F. Kennel, Pitch angle diffusion of radiation belt electrons within the plasmasphere, *J. Geophys. Res., 77*, 3455, 1972.

Lyons, L.R., and R.M. Thorne, Equilibrium structure of radiation belt electrons, *J. Geophys. Res., 78*, 2142, 1973.

Lyons, L.R., (1974) Pitch angle and energy diffusion coefficients from resonant interactions with ion cyclotron and whistler waves, *J. Plasma Phys., 12*, 417.

Peddie, N.W., International Geomagnetic Reference Field: The third generation, *J. Geomagn. Geoelectr., 34*, 309, 1982.

Northrop, T.G., *The Adiabatic Motion of Charged Particles*, Wiley, New York, 1963.

Roederer, J.G., *Dynamics of Geomagnetically Trapped radiation*, vol. 2, in *Physics and Chemistry of Space*, Springer-Verlag, 1970.

Summers, D. (2005), Quasi-linear diffusion coefficients for field-aligned electromagnetic waves with applications to the magnetsphere, *J. Geophys. Res., 110*, A08213, doi:10:1029/2005JA011159.

Ukhorskiy, A., B.J. Anderson, P.C. Brandt, and N.A. Tsyganeko (2006), Storm time evolution of the outer radiation belt: transport and losses, *J. Geophys. Res., 111*, A11S03, doi:10.1029/2006JA011690.

Varotsou, A., D. Boscher, S. Bourdarie, R.B. Horne, S.A. Clauert, and N. Meredith (2005), Simulation of the outer radiation belt electrons near geosynchronous orbit including both radial diffusion and resonant interaction with whistler mode chorus waves, *Geophys. Res. Lett., 32*, L19106, doi:10.1029/2005GL023282.

Vette, J.I. (1991), The AE-8 trapped electron model environment, *NSSDC/WDC-A-RS 91-24*, NASA Goddard Space Flight Center, Greenbelt, MD.

Walt, M., and W. MacDonald, The influence of the Earth's atmosphere on geomagnetically trapped particles, *Rev. Geophys., 2*, 543, 1964.

Zheng, Y., M.C. Fok, and G.V. Khazanov (2003), A radiation belt-ring current forecasting model, *Space Weather, 3*, 1013, doi:10.1029/2003SW00007.

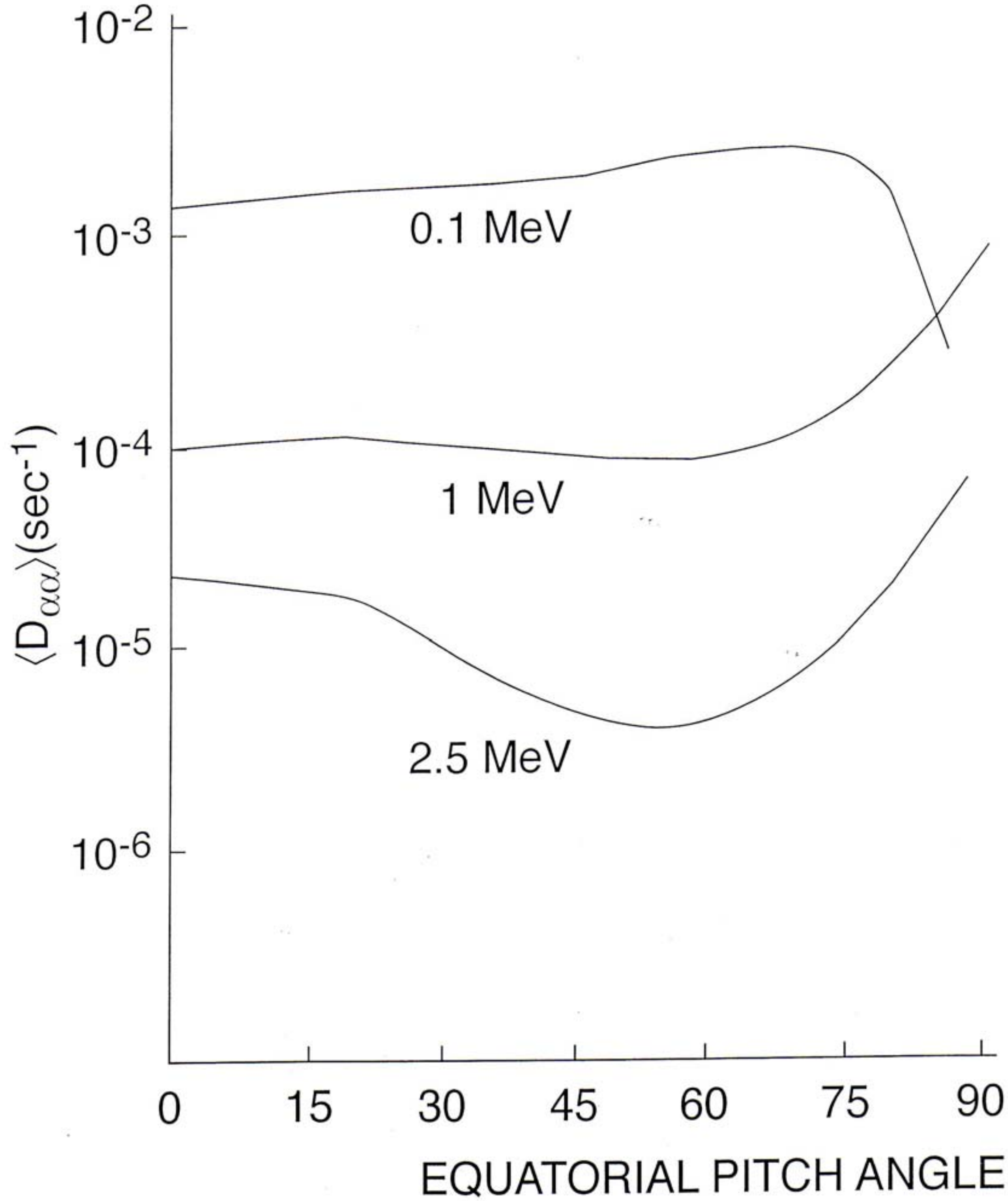
10-2
10-3
10-4
10-5
10-6
⟨Dαα⟩(sec-1)
0.1 MeV
1 MeV
2.5 MeV
0
15
30
45
60
75
90
EQUATORIAL PITCH ANGLE

Figure 1

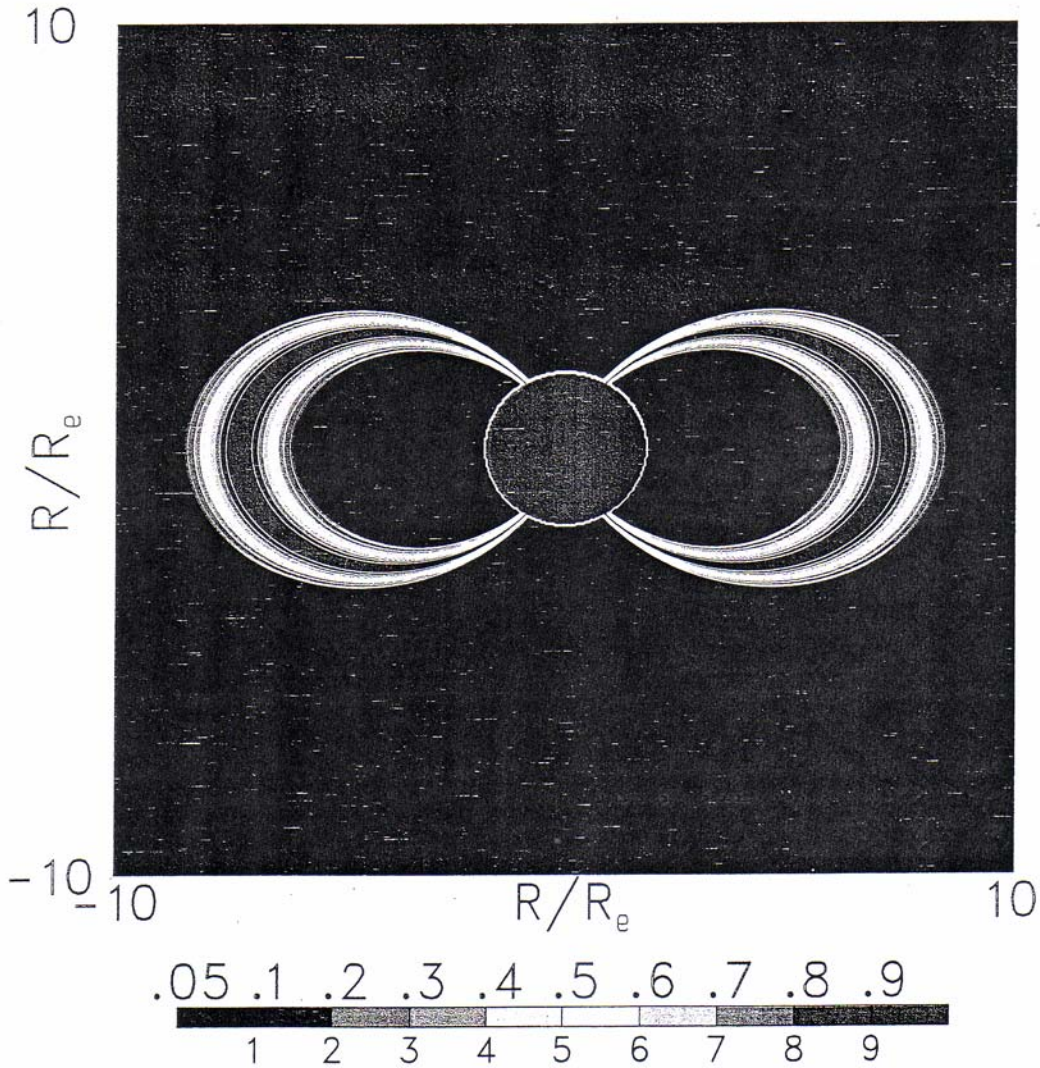

Figure 2

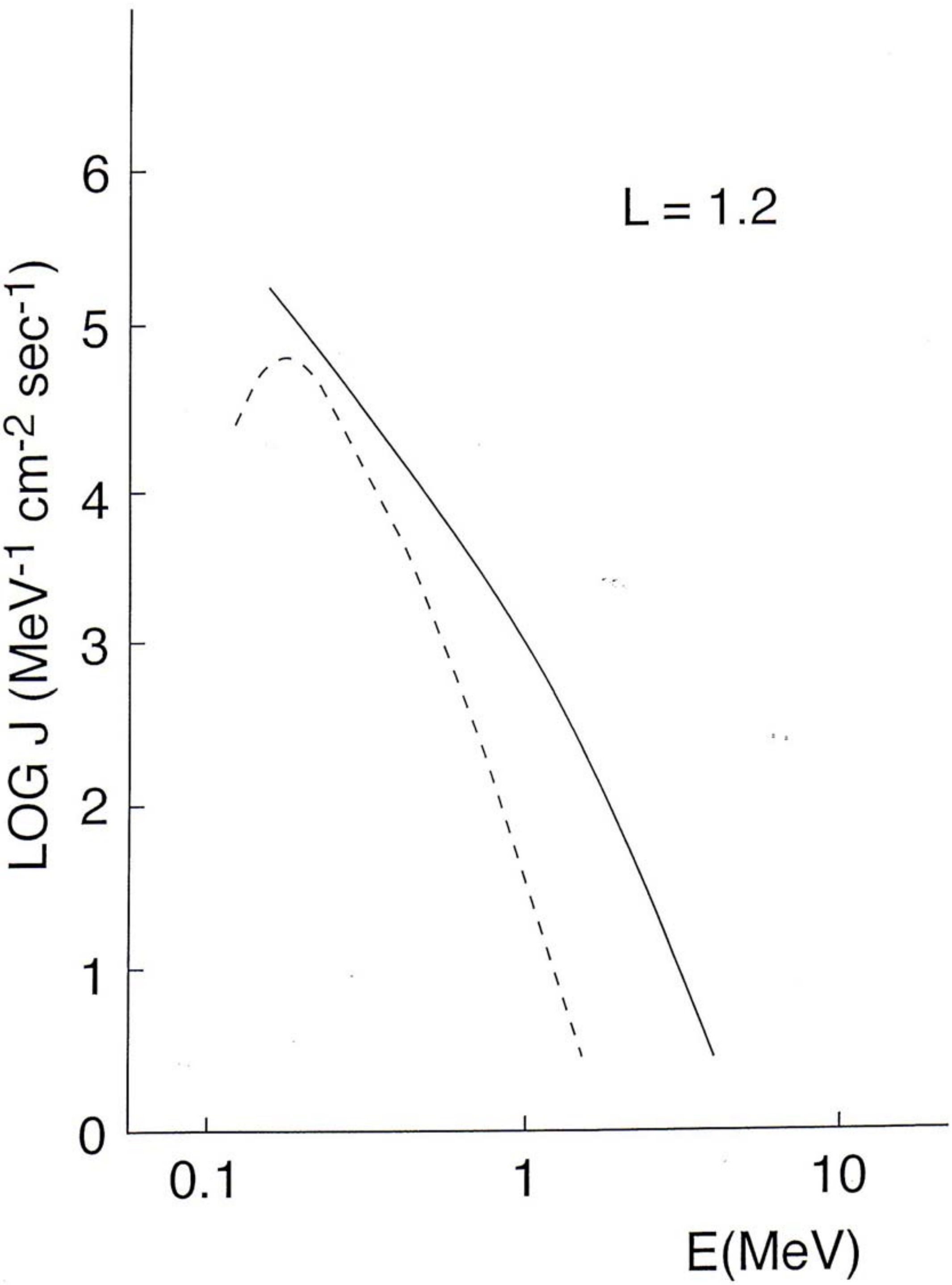

Figure 3

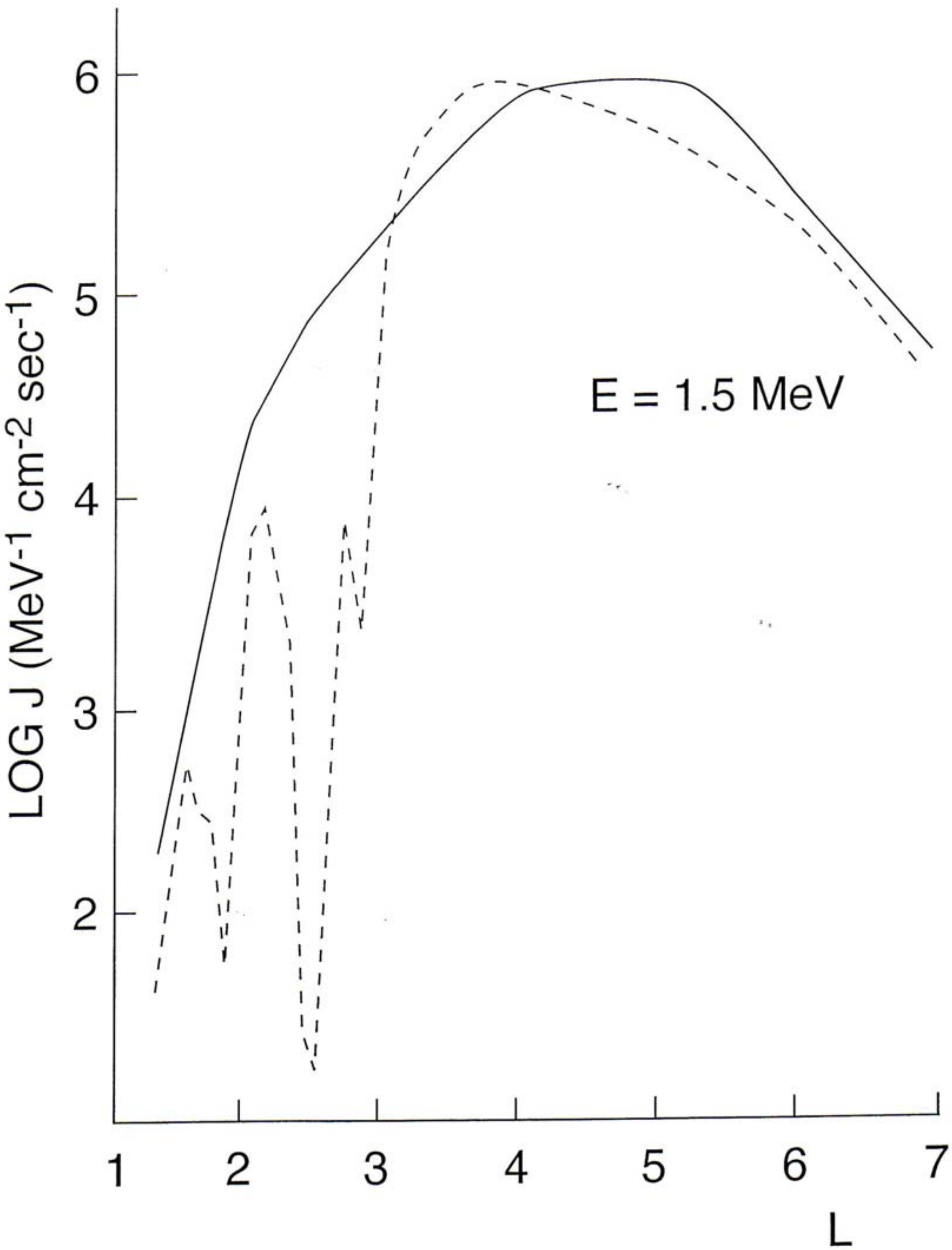

Figure 4

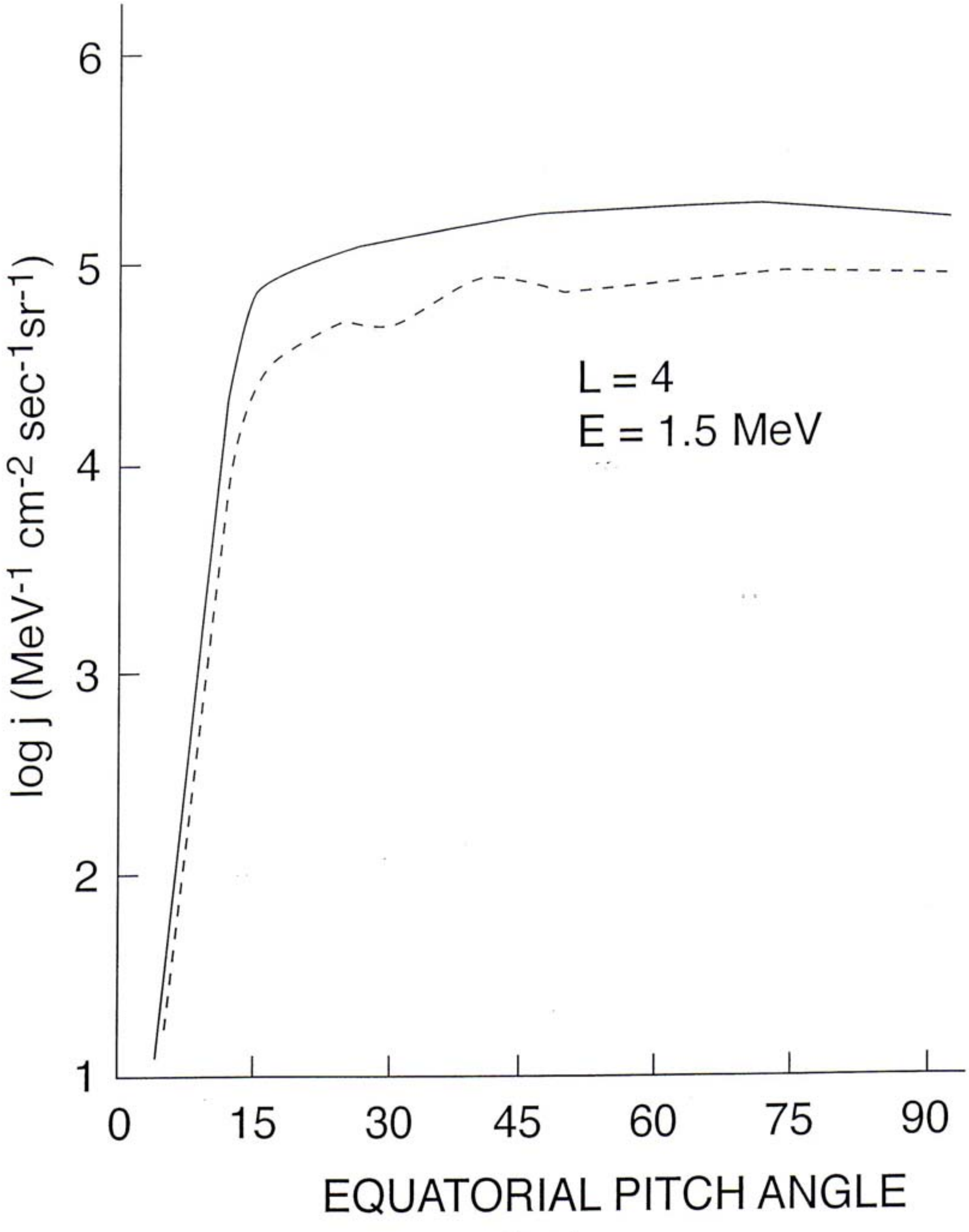
6
5
4
3
2
1
log j (MeV-1 cm-2 sec-1sr-1)
L = 4
E = 1.5 MeV
0
15
30
45
60
75
90
EQUATORIAL PITCH ANGLE

Figure 5